Optimal Output Trajectory Redesign for Invertible Systems

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I. Introduction

GIVEN a desired output trajectory, inversion-based techniques find input-state trajectories required to exactly track the output.[1,2] These inversion-based techniques have been successfully applied to the endpoint tracking control of multijoint flexible manipulators in Ref. 3 and to aircraft control in Ref. 4. The specified output trajectory uniquely determines the required input and state trajectories that are found through inversion. These input-state trajectories exactly track the desired output; however, they might not meet acceptable performance requirements. For example, during slewing maneuvers of flexible structures, the structural deformations, which depend on the required state trajectories, may be unacceptably large. Further, the required inputs might cause actuator saturation during an exact tracking maneuver, for example, in the flight control of conventional takeoff and landing aircraft.[5] In such situations, a compromise is desired between the tracking requirement and other goals such as reduction of internal vibrations and prevention of actuator saturation; the desired output trajectory needs to be redesigned.

Here, we pose the trajectory redesign problem as an optimization of a general quadratic cost function and solve it in the context of linear systems. The solution is obtained as an off-line prefilter of the desired output trajectory. An advantage of our technique is that the prefilter is independent of the particular trajectory. The prefilter can therefore be precomputed, which is a major advantage over other optimization approaches (see Ref. 6 for further references).

Previous works have addressed the issue of preshaping inputs to minimize residual and in-maneuver vibrations for flexible structures; see, for example, Refs. 6 and 7. Since the command preshaping is computed offline, in Ref. 8, the use of noncausal prefilters has been suggested—such noncausality is allowable since the command preshaping is computed off-line. Further, minimization of optimal quadratic cost functions has also been previously used to preshape command inputs for disturbance rejection in Ref. 9. All of these approaches are applicable when the inputs to the systems are known a priori. Typically, outputs (not inputs) are specified in tracking problems, and hence the input trajectories have to be computed. The inputs to the system are, however, difficult to determine for nonminimum phase systems like flexible structures. One approach to solve this problem is to 1) choose a tracking controller (the desired output trajectory is now an input to the closed-loop system) and 2) redesign this input to the closed-loop system. Thus, we effectively perform output redesign.[6] These redesigns are, however, dependent on the choice of the tracking controllers.[10] Thus, the controller optimization and trajectory redesign problems become coupled; this coupled optimization is still an open problem. In contrast, we decouple the trajectory redesign problem from the choice of feedback-based tracking controller. It is noted that our approach remains valid when a particular tracking controller is chosen. In addition, the formulation of our problem not only allows for the minimization of residual vibrations as in available techniques[6] but also allows for the optimal reduction of vibrations during the maneuver, e.g., the altitude control of flexible spacecraft.[9] We begin by formulating the optimal output trajectory redesign problem and then solve it in the context of general linear systems. This theory is then applied to an example flexible structure, and simulation results are provided.

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II. Problem Formulation

System Inversion for Exact Tracking

Consider a square system described by

$$\dot{x} = Ax + Bu; \qquad y = Cx$$

where $x \in \Re^n$, $u \in \Re^p$, and $y \in \Re^p$. The inversion approach[2] finds a bounded input-state trajectory that satisfies the preceding system equations and yields the exact desired output, i.e.,

$$\dot{x}_{\text{ref}} = Ax_{\text{ref}} + Bu_{ff}; \qquad y_d = Cx_{\text{ref}}$$

The inverse input-state trajectories can be described in terms of Fourier transforms as[1,11]

$$\begin{aligned} u_{ff}(j\omega) &= [C(j\omega I - A)^{-1}B]^{-1}y_d(j\omega) = G_y^{-1}(j\omega)y_d(j\omega) \\ x_{\text{ref}}(j\omega) &= [(j\omega I - A)^{-1}B]u_{ff}(j\omega) = G_x(j\omega)u_{ff}(j\omega) \end{aligned} \tag{1}$$

This Fourier-based inversion approach has been extended to nonlinear time-varying nonminimum phase systems in Ref. 2; however, we restrict our discussion to linear time-invariant systems.

Remark. We note two results. One, an inverse exists if the output and a certain number of its time derivatives are bounded. The number of time derivatives of the output that needs to be specified for an inverse to exist is well defined and depends on the relative degree of the system.[2,12] Second, for linear hyperbolic systems, if the inverse exists, then it is unique.[1,2]

Performance Criterion

Trajectory redesign seeks a compromise between the goal of tracking the desired trajectory and other goals such as reducing the magnitude of input and vibrations. We formulate this redesign problem as the minimization of a quadratic cost function of the type

$$\int_{-\infty}^{\infty} \Big\{u(t)^T Ru(t) + x(t)^T Q_x x(t) + [y(t) - y_d(t)]^T Q_y [y(t) - y_d(t)]\Big\}\, dt$$

where R, Q_x, and Q_y represent the weight on control input, states, and the error in output tracking, respectively.

Using Parseval's theorem, we rewrite our optimization problem in frequency domain as the minimization of the cost function

$$J = \int_{-\infty}^{\infty} \Big\{u(j\omega)^* Ru(j\omega) + x(j\omega)^* Q_x x(j\omega) + [y(j\omega) - y_d(j\omega)]^* Q_y [y(j\omega) - y_d(j\omega)]\Big\}\, d\omega \tag{2}$$

where the superscript $*$ denotes complex conjugate transpose.

Optimal Redesign of the Output

Our main result is given by the following lemma, which shows that the optimal output trajectory redesign can be described as a prefilter, which maps desired output trajectories y_d to the redesigned output trajectory y_{opt}. This prefilter G_f does not depend on the particular choice of desired trajectory and hence can be precomputed.

Lemma. The modified output trajectory y_{opt} is given by $y_{\text{opt}}(j\omega) = G_f(j\omega)y_d(j\omega)$, where

$$G_f(j\omega) = 1 - G_y\big[R + G_x^* Q_x G_x + G_y^* Q_y G_y\big]^{-1} \times \big[R + G_x^* Q_x G_x\big]G_y^{-1}$$

The modified input trajectory u_{opt} is given by $u_{\text{opt}}(j\omega) = u_{ff}(j\omega) + v_{\text{opt}}(j\omega)$, where $v_{\text{opt}}(j\omega) = G_v(j\omega)y_d(j\omega)$ and

$$G_v(j\omega) = -\big(R + G_x^* Q_x G_x + G_y^* Q_y G_y\big)^{-1} \times \big(R + G_x^* Q_x G_x\big)G_y^{-1} \tag{3}$$

Note that the dependence on $j\omega$ is not explicitly written for compactness.

Proof. Without loss of generality, we rewrite the input u as the sum of the feedforward input, $G_y^{-1}y_d$, found from inversion of the desired trajectory, and an arbitrary v:

$$u(j\omega) = u_{ff}(j\omega) + v(j\omega) = G_y^{-1}(j\omega)y_d(j\omega) + v(j\omega) \tag{4}$$

Substituting $x(j\omega) = G_x(j\omega)u(j\omega)$ and $y(j\omega) = G_y(j\omega)u(j\omega)$ along with the preceding Eq. (4) for u into the cost function given by Eq. (2), we obtain

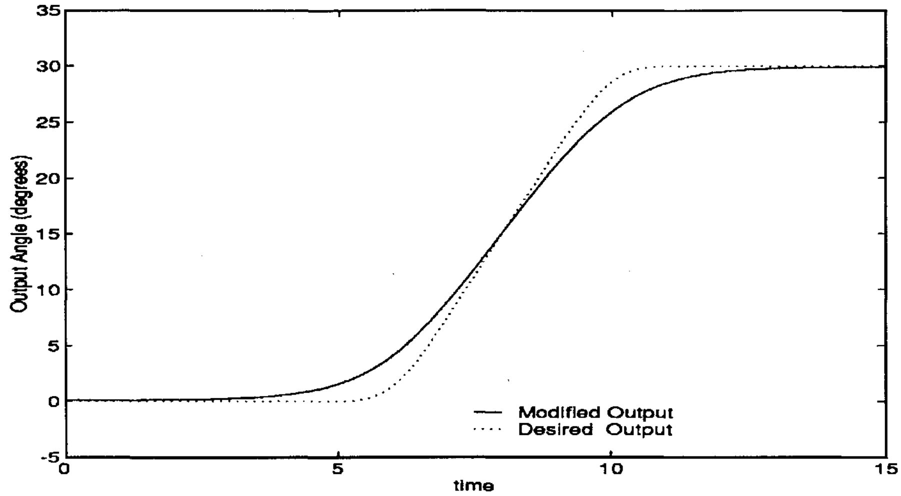

Fig. 1 Output redesign.

$$J = \int_{-\infty}^{\infty} \{[v + (R + G_x^* Q_x G_x + G_y^* Q_y G_y)^{-1} \times (R + G_x^* Q_x G_x) G_y^{-1} y_d]^* * (R + G_x^* Q_x G_x + G_y^* Q_y G_y) \times [v + (R + G_x^* Q_x G_x + G_y^* Q_y G_y)^{-1} \times (R + G_x^* Q_x G_x) G_y^{-1} y_d] + (G_y^{-1} y_d)^* [(R + G_x^* Q_x G_x) + (R + G_x^* Q_x G_x)^* * (R + G_x^* Q_x G_x + G_y^* Q_y G_y)^{-1} \times (R + G_x^* Q_x G_x)](G_y^{-1} y_d)\} \,\mathrm{d}w$$

Note that the cost function is quadratic in v. Therefore, the cost function is minimized by setting this quadratic term to zero, i.e., choosing $v(j\omega) = v_{\text{opt}}(j\omega) = G_v(j\omega) y_d(j\omega)$, where G_v is defined by Eq. (3) in the lemma. The choice of $v = v_{\text{opt}}$ defines the optimal input u_{opt} through Eq. (4) as

$$u_{\text{opt}}(j\omega) = [G_y^{-1}(j\omega) + G_v(j\omega)] y_d(j\omega) \tag{5}$$

The result follows from $y_{\text{opt}}(j\omega) = G_y(j\omega) u_{\text{opt}}(j\omega) = [1 + G_y(j\omega) G_v(j\omega)] y_d(j\omega)$. □

III. Example

Consider a flexible structure consisting of two freely rotating disks connected by a thin shaft. A motor is attached between the connecting shaft and one of the disks. Input to the system is torque τ provided by a dc motor, and the outputs are the angular rotations of the two disks θ_1 and θ_2. These angular rotations are measured using potentiometers. The transfer function of an experimental system, which includes the rigid-body mode and one flexible mode, was obtained using a HP3562A Dynamic Signal Analyzer as

$$\frac{\theta_1}{\tau} = \frac{1.8139 s^2 + 0.3077 s + 6.1041}{s^4 + 0.2765 s^3 + 6.1041 s^2}$$
$$\frac{\theta_2}{\tau} = \frac{0.27 s^2 - 0.1187 s + 6.1041}{s^4 + 0.2765 s^3 + 6.1041 s^2} \tag{6}$$

With the state vector x chosen as $x = [\theta_1\ \theta_2\ \dot{\theta}_1\ \dot{\theta}_2]^T$, the system equations can be represented in state–space form as $\dot{x} = Ax + Bu$, i.e.,

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -3.1555 & -0.1640 & 3.1555 & 0.3845 \\ 0 & 0 & 0 & 1 \\ 2.8956 & -0.0899 & -2.8956 & -0.1124 \end{bmatrix} \times \begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1.8139 \\ 0 \\ 0.27 \end{bmatrix} \tau$$

with $y = \theta_2 = [0\ 1\ 0\ 0]x$. The control objective is to track the angular rotation θ_2 of the disk that is farthest away from the motor (see Fig. 1 for the desired output trajectory).

The relative degree of a single-input/single-output linear system is the number of zeros at infinity.[12] For our system, with the torque as input and θ_2 as output, the transfer function has four poles and two finite zeros [see Eq. (6)]. Thus, the number of zeros at infinity are two, and hence the relative degree is two. This implies that the second derivative of the desired output, i.e., the desired angular acceleration profile of the output, uniquely determines the required input-state trajectory and the resulting structural vibration, $\theta_1 - \theta_2$.[2] If the internal vibrations are to be reduced, then we have to relax the exact tracking requirement. Similarly, to reduce the required input amplitudes we have to compromise exact tracking. This tradeoff can be represented as the minimization of a general quadratic cost function (Sec. II) of the form

$$\int_{-\infty}^{\infty} \{u(t)^T R u(t) + x(t)^T Q_x x(t) + [y(t) - y_d(t)]^T Q_y [y(t) - y_d(t)]\} \,\mathrm{d}t$$

where $R = r$, $Q_y = q_y$, and

$$Q_x = q_x \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The scalars r, q_x, and q_y represent the relative weight on the reduction of inputs, structural vibrations, and tracking errors, respectively. To reduce the vibrations and control inputs, we choose $r = 1$, $q_x = 5000$, and $q_y = 1$ in our simulations. Figure 1 shows the modification for a desired trajectory— about 10% of the final slew angle. The maximum magnitude of the required input, however, is reduced by 60%, and the corresponding structural vibration, $\theta_1 - \theta_2$, is reduced by 20% (compared with results from exact tracking of the initial desired trajectory).

IV. Conclusion

We formulated and solved the trajectory redesign problem in the context of linear invertible systems, including nonminimum phase systems. Thus, we provide a systematic approach to an optimal tradeoff between tracking desired trajectory and other goals such as vibration reduction and reduction of required inputs. The approach was applied to an example flexible structure, and simulation results were presented. Future work will address trajectory redesign for nonlinear systems.

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Instrumentation Analysis Using Parameter Estimation of Simulated Data

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Introduction

PARAMETER estimation is an important tool in the development of aerodynamic models for aircraft. By processing data recorded during flight tests, parameter estimation techniques can calculate estimates of the aircraft stability and control derivatives. Software designed to perform parameter estimation is typically tested on simulated data created from a known set of parameters before it is used to examine actual flight data. This research focuses on expanding the role of simulated data beyond estimator validation to examine a wide variety of parameter estimation issues. The goal of the project is to improve the effectiveness of aircraft model development from flight test data.

A previous paper described an analysis package developed to assist with the preliminary analysis of maneuver design, flight conditions, instrumentation, data processing, and model structure.[1] For this research, the analysis package was used to examine the instrumentation system of one of North Carolina State University's unmanned air vehicles. To provide realistic simulated data, the aircraft was modeled using an existing simulation environment.[2] The simulation includes an elaborate model of the instrumentation system, incorporating the effects of sensor position, sensor dynamics, random noise, analog filtering, and digital storage for each transducer in the system.

The analysis package employs the output error approach to estimate the parameters in a state space model of the aircraft dynamics. Output error has been used for aircraft analysis more than any other parameter estimation technique.[3,4] There are a number of newer and more advanced techniques available now,[5–7] but output error was chosen because of its proven track record for consistent and reliable results. The analysis package was used to study a wide variety of instrumentation issues, but the most significant results involve the analysis of the angle of attack transducer.

Alpha Vane Transducer

The angle of attack (α) measurement is one of the primary observations for an aircraft longitudinal dynamic model. The unmanned air vehicle measures the angle of attack with an alpha vane mounted on a boom in front of the aircraft. The dynamic response of the vane is dependent on the vane sizing, the moment of inertia, and the friction of the system. The response of the vane can be modeled as a second-order low-pass filter of the form

$$G_{\text{alpha}}(s) = \frac{\omega_n^2}{s^2 + 2\omega_n \zeta s + \omega_n^2}$$

The natural frequency ω_n and damping ratio ζ of the vane are estimated to be 5 Hz and 0.7, respectively. In the simulation the continuous transfer function is approximated as a second-order difference equation.

Preliminary Results

The first experiments with the analysis tools attempted to obtain a simple longitudinal model of the aircraft from a set of data recorded on the flight simulator. The results were disappointing. Table 1 shows the estimates and errors for the four main parameters. Large errors in the critical parameter $C_{N\alpha}$ were observed for a wide variety of input maneuvers. To identify the problem, different portions of the instrumentation model were deactivated in the flight simulator. The data collected without the sensor dynamic models showed a noticeable improvement in the estimates. Further analysis identified that the alpha vane was responsible for the inaccuracies, because the other longitudinal sensors (pitch rate and vertical accelerometer) have very fast response times. The lag introduced by the alpha sensor dynamic model causes a delay in the data that interferes with the output error calculation of the stability derivatives.

Several corrections for this problem were considered. A faster approximation of the vane dynamics could be used, but it might be impractical to construct a transducer to match these dynamics. Alternatively, the output error model could be modified to include the dynamics of the vane as states, but this would increase the model complexity and potentially degrade the performance of the estimator. The most desirable correction would be to remove the lag from the angle of attack signal. An effort was made to perform this correction in the postprocessing of the recorded data, but the relatively slow sampling rate (25 Hz) makes this impractical. The remaining option was to design an analog lead circuit for the instrumentation system to modify the signal before it is digitally sampled and recorded by the flight computer.

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